

Measuring the re-absorption cross section of a Magneto-Optical Trap

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Magneto-Optical Traps have been used for several decades. Among fundamental mechanisms occurring in such traps, the magnitude of the multiple scattering is still unclear. Indeed, many experimental situations cannot be modeled easily, different models predict different values of the re-absorption cross section, and no simple experimental measurement of this cross section are available. We propose in this paper a simple measurement of this cross section through the size and the shape of the cloud of cold atoms. We apply this method to traps with a configuration where theoretical values are available, and show that the measured values are compatible with some models. We also apply the method to configurations where models are not relevant, and show that the re-absorption is sometimes much larger than the usually assumed value.

I. INTRODUCTION

The Magneto-Optical Trap (MOT) is today an essential tool in experimental atomic physics. A MOT produces ultracold atoms which can be used in numerous experiments: MOTs are used – alone or with further cooling steps – to study quantum chaos [1], Anderson localization [2] or plasma instabilities [3, 4], to produce ultra precise atomic clocks [5], to obtain cold molecules [6], and many other studies.

In most experiments, an accurate knowledge of the detailed physical mechanisms occurring in a MOT is not necessary. For example, it is well known that the atomic cloud produced by a MOT becomes unstable for some parameters, but it is also well known that in most cases, the alignment of the laser beams has just to be adjusted to remove these instabilities. However, recent studies have shown that understanding the detailed physics of the MOT could be interesting, because the MOT appears to belong to the large family of complex systems described by Vlasov-Fokker-Planck (VFP) equations [4]. Because MOT experiments are relatively simple to implement within this family, they appear as a good candidate for a model system in this area. And anyway, understanding in detail the physics of MOT would allow to understand what is observed in the experiments.

The mechanisms allowing to obtain a cloud of cold atoms with a MOT are well known. They result from the interactions of the atoms with the laser light. The basic interaction is the absorption of a photon by an atom, followed by a spontaneous emission process. Depending in particular on the cloud density, the emitted photon can possibly undergo several cycles of absorption/emission before escaping the cloud: it is the multiple scattering. The final cloud of cold atoms is the result of the equilibrium between the trapping force (induced by the exchange of momentum during the diffusion cycle), the shadow effect force (induced by the absorption of the beam through the cloud), and the multiple

scattering force. The amplitude of the forces depends in particular on the absorption cross section σ_L and on the re-absorption cross section σ_R . Knowing these quantities is thus essential to model accurately a MOT. Moreover these cross sections play a key role in the analogy with plasma physics because the effective charge involved in the coulomb-like interaction between cold atoms depends on their ratio [7].

The detailed mechanisms involved in a MOT are rather complex. Taking into account the exact distribution of the atomic levels and the interaction of the atoms with all the laser beams would lead to a very complex set of equations. Thus all the existing models deal with approximations which lead to different predictions of the cross section values. This is not a big deal concerning σ_L because simple absorption measurements lead to this quantity. On the contrary, no simple way to measure σ_R has been proposed until now. So theoretical predictions have never been validated by experimental measurements, and the σ_R values found today in the literature are still questionable.

We propose in this paper a method to measure the re-absorption cross section in a magneto-optical trap. We apply this method to a Cesium trap and obtain values of σ_R for different sets of laser parameters. We compare these values to different models.

II. THEORY

We consider here a MOT in the usual $\sigma^+ - \sigma^-$ configuration. Each of the three arms of the trap consists in a pair of counter-propagating laser beams characterized by their intensity and their frequency. A pair is obtained by retro-reflection of an incident beam. The beam intensities are respectively I_+ for the incident beam and I_- for the retro-reflected one, while their frequency is given through the detuning Δ between the laser frequency and the atomic transition. We do not need to detail the energetic structure of the cold atoms in order to understand the steady state of the cloud. Indeed, we adopt a global formalism in term of cross sections.

The stationary cloud of cold atoms results from the

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equilibrium between three forces acting on the atoms. These forces have been expressed in numerous studies, with more or less approximations [4, 7–10]. However, all these models deal with the same variables and the same parameters. The first force is the trapping force produced by the laser beams and the Zeeman shifts induced by the magnetic field. It is a restoring force, characterized by the spring constant κ . The two other forces are collective forces. The shadow effect force is due to the absorption of the laser beams all along the cloud, leading to a local imbalance of the laser beam intensities. Thus this force depends on the absorption cross section σ_L . Finally, the multiple scattering force is induced by additional scattering of photons, and so depends on the re-absorption cross section σ_R . The first two forces compress the cloud of atoms, while the latter causes it to expand. The size of the obtained atomic cloud is thus the result of an equilibrium between these three forces.

To study the equilibrium, we use an approach similar to that in [7] which assumes that the temperature of the cloud is zero. The main assumption in this model is that a photon is re-scattered at most once before escaping the cloud. Contrary to [7], we consider the usual anti-Helmholtz configuration for the coils creating the magnetic field, as mentioned previously. Such a field is zero at the point defined as the center of the trap and it can be assumed to be linear along each direction. We take into account the fact that the magnetic field gradient along the coil axis is twice that along the perpendicular directions. So, the spherical symmetry used in [7] is broken, and the cloud shape must be modeled as an ellipsoid.

The determination of the stationary density n does not need the knowledge of the forces. Indeed, the collective forces depend on the shape of the cloud, while their divergences do not. The vanishing of the divergence of the total force (which is zero at equilibrium) gives us a constant atomic density. The results from [11], where a more general anisotropic configuration has been considered, can be applied to the present situation and give:

$$n = \frac{2c\kappa}{3I_+ \sigma_L^2 (S - 1)} \quad (1)$$

where $S = \sigma_R/\sigma_L$ is the cross section ratio and c is the speed of light. Note that as the quadrupolar magnetic field is taken into account, this expression differs slightly from that in [7, 8]. We can improve this description of the MOT by calculating the expression of the three forces. The trapping force has a usual form and takes into account the anisotropy of the magnetic field gradient. The shadow effect force has the same expression as in [7], the absorption is assumed to be linear. The net multiple scattering force is calculated as the sum over the cloud of all the coulomb-like atom-atom interactions associated with a scattering process. In [11], it is shown how to calculate this force for an ellipsoidal cloud, with half-widths L_{\parallel} and L_{\perp} along the coil axis and in the transverse plane respectively. A geometric parameter A , depending only on the ellipticity $\varepsilon = L_{\perp}/L_{\parallel}$ of the cloud, appears in the

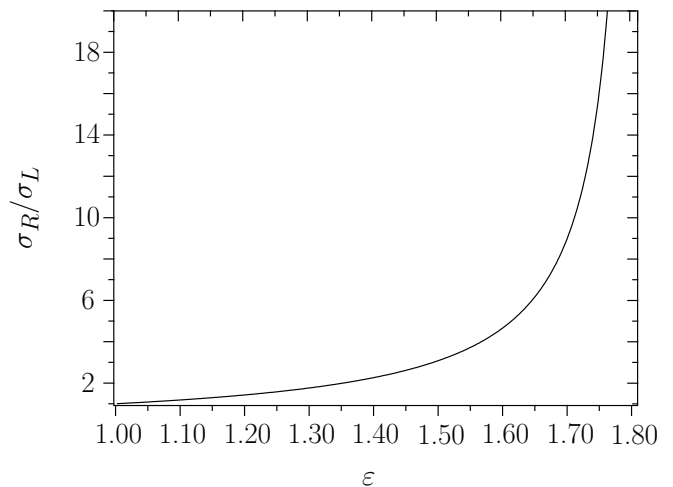


Figure 1: Evolution of the cross section ratio inside the predicted range of ellipticity values.

expression of the force:

$$A = \frac{\varepsilon^2}{\varepsilon^2 - 1} \beta \quad (2)$$

$$\beta = \begin{cases} 1 - \frac{1}{\sqrt{1 - \varepsilon^2}} \ln \left| \frac{1 + \sqrt{1 - \varepsilon^2}}{1 - \sqrt{1 - \varepsilon^2}} \right| & \text{for } \varepsilon^2 > 1 \\ 1 - \frac{1}{\sqrt{\varepsilon^2 - 1}} \arcsin \left(\sqrt{\frac{\varepsilon^2 - 1}{\varepsilon^2}} \right) & \text{for } \varepsilon^2 < 1 \end{cases}$$

The equilibrium of the forces gives another condition on this parameter :

$$A = \frac{1}{2} \left(1 - \frac{1}{3S} \right) \quad (3)$$

For the sake of completeness, the equilibrium of the forces also gives us access to the cloud displacement due to the shadow effect in our asymmetric configuration. This displacement is global, and depends only on the shadow effect: it gives no information about multiple scattering.

From the previous equations, it is easy to find that ε is bounded. First, the atomic density is a positive quantity so that, in eq. 1, σ_R has to be larger than σ_L ($S > 1$). It follows from eq. 3 that $A > 1/3$. From the same equation, we also find that the maximum value of A is $1/2$ (when $S \rightarrow +\infty$). This leads to:

$$1 < \varepsilon < 1.81 \quad (4)$$

Thus, the atomic cloud shape of a usual MOT appears to be always oblate. A spherical cloud corresponds to a infinite density, and so is not a physical solution. We also point out an upper limit to the ellipticity which corresponds to $\sigma_R \gg \sigma_L$.

Fig. 1 shows the cross section ratio versus the ellipticity for all possible values: the more elongated the cloud, the more probable the re-absorption. This result is quite

Table I: Comparison of theoretical expressions for the cross section ratio for the two limit cases $\Omega \gg |\Delta| \gg \Gamma$ and $|\Delta| \gg \Omega \gg \Gamma$.

ref.	$\Omega \gg \Delta $	$\Omega \ll \Delta $	method
[8]	1	2	phenomenological calculation
[9]	$\frac{\Delta^2}{2\Gamma^2}$	$\frac{3\Omega^2}{\Gamma^2}$	<i>two-level atom</i>
			curve fitting / DAP
[10]	$\frac{\Delta^2}{3\Gamma^2}$	$\frac{\Omega^2}{2\Gamma^2}$	<i>two-level atom</i>
			analytical expression / DAP
[4]	$\frac{\Delta^2}{6\Gamma^2}$	$\frac{\Omega^2}{2\Gamma^2}$	<i>two-level atom</i>
			analytical expression / DAP
			<i>three-level atom</i>

different from the one predicted for the temperature-limited regime in which the ellipticity of the cloud is constant and equal to $\sqrt{2}$ [9]. In the multiple scattering regime, the ratio S and the cloud ellipticity depend *a priori* on the laser parameters.

As pointed out above, the calculation of the re-absorption cross section as a function of the laser parameters is rather complex. The atom must be modeled with a particular atomic structure from which the overlap between the emission and the absorption spectra of a cold atom can be calculated. Several authors figured out theoretical expressions of σ_R in the context of Doppler cooling theory [4, 7–10], but with different approaches. In [7], a fit of single cloud size measurements is performed for only one set of parameter values. [8] performs an approximated analytical calculation, while [4, 9, 10] use the dressed atom picture (DAP) [12]. All consider a two-level atom, except [4] who considers a three-level atom. These calculations take into account the saturation by the two counter-propagating beams, requiring to introduce the total Rabi frequency $\Omega = \Gamma \sqrt{(I_+ + I_-)/2I_{sat}}$, with I_{sat} the saturation intensity ($I_{sat} = 1.1 \text{ mW/cm}^2$ for Cs) and Γ the natural width of the transition.

Within the DAP, the secular approximation limits the range of parameter values to $\Omega^2 + \Delta^2 \gg \Gamma^2$. But even with such a simplification, the expression of σ_R remains heavy. Table I summarizes the expressions obtained by these previous studies, in the two limit cases where the laser intensity is much larger than the detuning and vice versa. On the one hand, these two situations ($|\Delta| \gg \Omega \gg \Gamma$ and $\Omega \gg |\Delta| \gg \Gamma$) give simple asymptotic expressions. And on the other hand, they represent usual experimental parameters. Note that the values of parameters in [7] do not fit these two limit cases, and the results from [8] do not seem to be relevant. The expressions derived from the DAP give the same dependence on the laser parameters, but with different numerical factors.

III. MEASUREMENTS

Equations 2 and 3 link the cross section ratio with the ellipticity. As ellipticity can be measured experimentally, we have a way to determine experimentally σ_R , and to compare this measurement to the theoretical predictions.

Our experimental set-up is described in [13]: we use a usual MOT in which each arm is formed by the retro-reflection of an incident beam. To guarantee repeatability and reproducibility of the measurement, some cares have to be taken. Indeed, the MOT laser beams can create interference patterns on the cloud. So, its shape is not symmetric and its density is not homogenous. Moreover, the beam relative phase fluctuations make the patterns to move randomly in time. To avoid these effects, we modulate the relative phases of the beams at a frequency larger than 1 kHz. This modulation is faster than the collective atomic response timescale. In addition, the shape of the cloud is very sensitive to the alignment of the beams. We improve the beam alignment to better than 0.1 mrad by observing the cloud and adjusting its symmetry.

In order to measure the ellipticity, we record the cloud fluorescence with a CCD camera, the optical axis of which is perpendicular to the coil axis. In that way, we obtain pictures showing a 2D projection of the ellipsoid. The pictures are fitted on a 2D gaussian, giving us the semi-axes L_{\parallel} and L_{\perp} of the ellipsoid. A typical experimental measurement consists in recording the cloud sizes as a function of Δ , and repeating this sequence for different values of Ω . For each set of parameters, we record 10 pictures in order to improve the precision and to evaluate the standard deviation.

Note that applying this method for very dilute or very dense clouds leads to large measurement uncertainties. Indeed, this method requires a good signal to noise ratio to monitor the cloud with a camera, which is not the case for dilute clouds, i.e. for large detunings or low laser intensities. On the other hand, in an asymmetric configuration, a thick cloud is displaced by the shadow effect. Simple observations show that the shadow effect degrades also the shape of the cloud, typically when the laser is tuned close to resonance. This issue does not exist for a MOT obtained with six independent beams.

We first check the predicted range of ellipticity values. We measure the cloud sizes for almost 120 different values of (Δ, Ω) . Fig. 2 shows all the measured values and how often they have been measured. The result is in good agreement with the theoretical range: about 90 % (if we consider error bars) of the measured ellipticities are inside the predicted range. No prolate cloud is observed. Furthermore, most of the values over the limit correspond to pictures recorded with a low signal to noise ratio, i.e. for which the measurement uncertainty is probably underestimated.

Fig. 3 shows the typical evolution of the ellipticity as a function of the detuning. In this example, the minimum value of the ellipticity is 1.4 and is measured around $\Delta = -6.5\Gamma$. Points over the 1.81 limit are observed for $|\Delta| \lesssim$

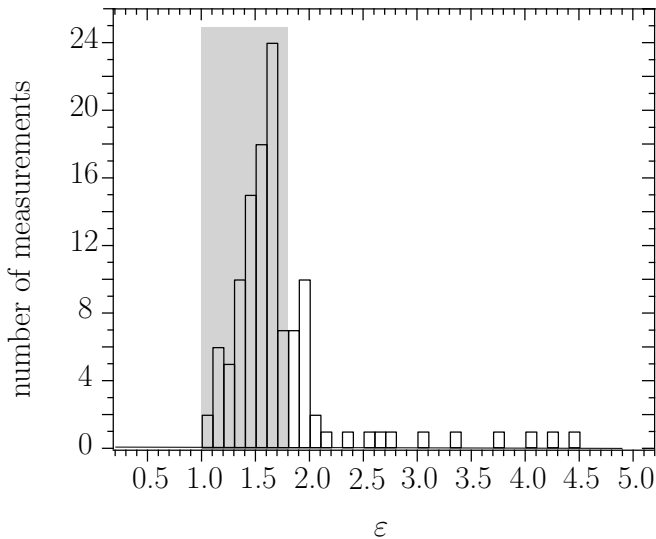


FIGURE 2: Measurements of the cloud ellipticity for 117 different values of the MOT parameters. The theoretical range is shown in gray.

3Γ , but as explained above, errors bars have probably been underestimated for these measurements.

Strictly speaking, the two limit cases considered in Table I cannot be satisfied with our experimental parameter values. However, for large enough Δ values, we approach the conditions $|\Delta| \gg \Omega \gg \Gamma$. Moreover, as the measurement quality is poor for very large detunings, we retain only intermediate values for our determination of ε . For example, in the case of Fig. 3, the estimation of the cross section ratio is done in the range $-8.8\Gamma < \Delta < -5.5\Gamma$. In this case, we have $\Omega \simeq \Gamma I_+ / \sqrt{2} I_{sat} \simeq 2.5\Gamma$ and $|\Delta| \gtrsim 2\Omega$. Note that the secular limit is satisfied because $\Omega^2 + \Delta^2 > 35\Gamma^2$. In this interval, the ellipticity is quite constant, as predicted, and we get $\varepsilon = 1.51 \pm 0.08$, i.e. $\sigma_R = 3.2\sigma_L$. This value is in good agreement with [4, 10] which gives the theoretical value S_{th} of 3.1.

Table II summarizes the measurements obtained in three different configurations. In each case, the measurement and the predicted value of S are very similar, with a relative difference of less than 5%. Unfortunately, despite the good precision on the ellipticity, the error on the cross section is significant. This is due to the non-linearity of the relation between these two quantities (Fig. 1). Moreover, we get asymmetric errors because the derivative is also non-linear. As a consequence the smaller the ellipticity, the better the precision.

Of course, the possibility to measure σ_R is particularly interesting for parameters where no theoretical predictions are available, or where theoretical predictions are questionable because of the approximations. It is in particular the case for smaller detunings and intensities. Fig. 4 shows the evolution of the cross section ratio for $-7.5\Gamma < \Delta < -1.8\Gamma$ and $I_+ \simeq 2.8I_{sat}$. It is interesting to note that S exhibits a maximum around $\Delta = -5\Gamma$, with values larger than 5. These values are rather high

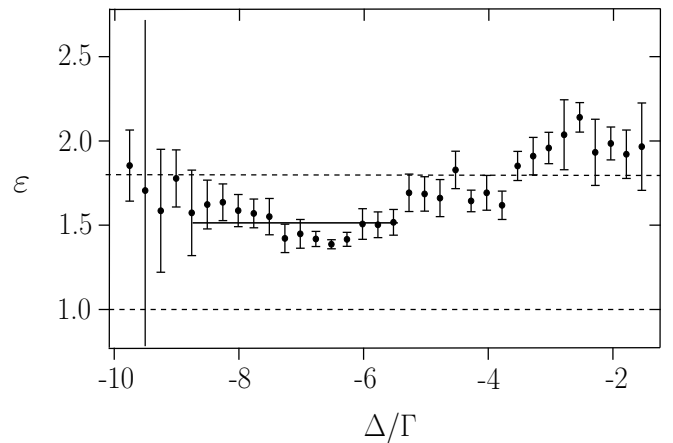


Figure 3: Evolution of the ellipticity ε versus the laser detuning Δ for $I_+ \simeq 3.6I_{sat}$. The dashed lines show the theoretical limits of ellipticity values ($1 < \varepsilon < 1.81$). The error bars represent the standard deviation from the 10 pictures.

Table II: Comparison of the experimental determination of S with the predictions. For different intensity values, we give the averaged ellipticity ε measured for a range of large detunings, the deduced cross section ratio S and the theoretical value S_{th} from [4, 10].

Ω^2/Γ^2	detuning domain	ε	S	S_{th}
4.5	$[-7.0\Gamma, -5.6\Gamma]$	1.39 ± 0.05	$2.2^{+0.3}_{-0.3}$	2.3
7.0	$[-8.8\Gamma, -5.5\Gamma]$	1.51 ± 0.08	$3.2^{+1.2}_{-0.7}$	3.1
9.4	$[-9.2\Gamma, -5.5\Gamma]$	1.61 ± 0.09	$4.9^{+4.1}_{-1.6}$	4.7

compared to those used in the literature but they are consistent with eq. 1. Close to the resonance, the ratio varies between 1 and 2, values which lead to a dense cloud as expected for these detunings. Whereas far from resonance the ratio is a little bit larger than 2 but the density is small due to a weak restoring.

IV. CONCLUSION

In this paper, we propose a method to determine experimentally the re-absorption cross section σ_R in a cloud of cold atoms. To our knowledge, it is the first method which allows to measure σ_R for a large set of MOT parameter values. The method is non-destructive and based on ellipticity measurements of the atomic cloud. The re-absorption cross section can be measured very easily. The results of the measurements are in good agreement with the calculation done with the dressed atom picture in the limit studied here (large detunings and intermediate intensities). We also make measurements for MOT parameters for which no theoretical predictions have been

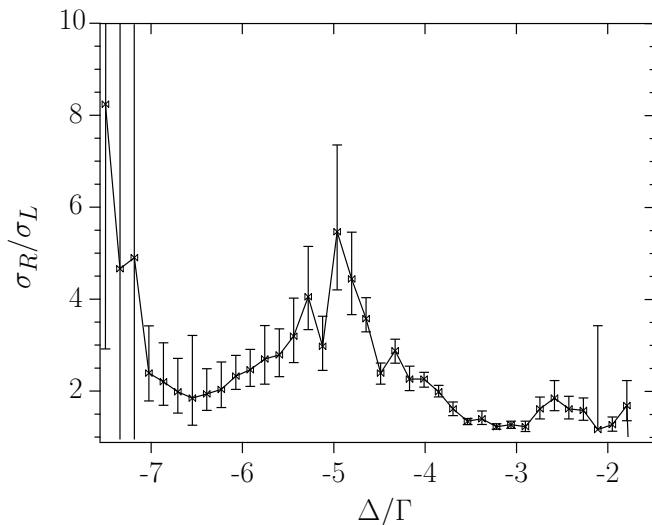


FIGURE 4: Evolution of the cross section ratio versus Δ for $I_+ \simeq 2.8I_{sat}$.

done. The measurements show that the cross section ratio is underestimated in the literature in this case. The precision of this method can be as high as desired, it requires only more acquisitions. A good precision is needed especially when an important re-absorption is expected. Measuring the cross section can be very useful to improve our description of the re-absorption in future studies on spatio-temporal dynamics of the atomic cloud.

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